Can Heavy WIMPs Be Captured by the Earth?

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ABSTRACT

If weakly interacting massive particles (WIMPs) in bound solar orbits are systematically driven into the Sun by solar-system resonances (as Farinella et al. have shown is the case for many Earth-crossing asteroids), then the capture of high-mass WIMPs by the Earth would be affected dramatically because high-mass WIMPs are captured primarily from bound orbits. WIMP capture would be eliminated for $M_x > 630\,\mathrm{GeV}$ and would be highly suppressed for $M_x \gtrsim 150\,\mathrm{GeV}$. Annihilation of captured WIMPs and anti-WIMPs is expected to give rise to neutrinos coming from the Earth's center. The absence of such a neutrino signal has been used to place limits on WIMP parameters. At present, one does not know if typical WIMP orbits are in fact affected by these resonances. Until this question is investigated and resolved, one must (conservatively) assume that they are. Hence, limits on high-mass WIMP parameters are significantly weaker than previously believed.

Subject headings: asteroids – elementary particles – dark matter – Galaxy: halo

1. Introduction

If weakly interacting massive particles (WIMPs) comprise the dark matter, they would have a local density $\rho_x \sim 0.3\,\mathrm{GeV\,cm^{-3}}$. They would be potentially detectable in several ways including directly in underground detectors, from WIMP-anti-WIMP annihilations in the Galactic halo, and inidrectly from neutrinos produced by annihilations of WIMPs captured by the Earth and the Sun (e.g., Jungman, Kamionkowski, & Griest 1996).

WIMP capture by the Sun was first discussed in the context of a proposed WIMP solution to the solar-neutrino problem (Press & Spergel 1985; Faulkner &

Gilliland 1985). Subsequently, several workers realized that if both WIMPs and anti-WIMPs were captured by the Sun (Silk, Olive, & Srednicki 1985; Gaisser, Steigman, & Tilav 1986; Srednicki, Olive, & Silk 1987; Griest & Seckel 1987) or Earth (Freese 1986; Krauss, Srednicki, & Wilczek 1986; Gaisser et al. 1986), they might annihilate into neutrinos which would then pass directly through these boides and into neutrino detectors near the surface of the Earth. The failure to detect neutrinos coming from the centers of the Sun and Earth could then place constraints on the WIMP mass M_x and cross section σ_x.

Because the Sun's gravitational potential is much deeper than the Earth's, it initially appeared as though the Sun would be so much more effective at capture that the annihilation signal from the Sun would always be much larger than from the Earth. However, Gould (1987) showed that there were "resonant enhancements" in capture by the Earth whenever the WIMP mass M_x was close to the mass M_A of an element A that is common in the Earth. Specifically, for hard-sphere cross sections σ_{xA}, the capture rate is given by (see also Gould 1992)

$$C = \frac{N_A \sigma_{xA}}{\beta_-} \int_0^{v_{\rm cut}} d^3u (v_{\rm cut}^2 - u^2) \frac{f(\mathbf{u})}{u} \tag{1}$$

where

$$v_{\rm cut}^2 \equiv \beta_- v_{\rm esc}^2, \qquad \beta_\pm \equiv \frac{4M_x M_A}{(M_x \pm M_A)^2}, \tag{2}$$

$f(\mathbf{u})$ is the distribution of velocities $\mathbf{u}$ of WIMPs relative to the Earth in the terrestrial neighborhood but away from the Earth's potential, N_A is the total number of atoms A in the Earth, and $v_{\rm esc} \sim 13\,{\rm km\,s^{-1}}$ is the escape velocity at the position of the atoms. In principle, one should allow for a range of escape velocities of the atoms, but for the case of iron in the Earth (of interest here), Gould (1987) showed that the error induced by simply using the average escape velocity is small.

In the present context, it is very important to note that $v_{\rm cut}$ is the maximum velocity for which an ambient WIMP can be captured by the Earth. If $M_A \sim M_x$, then $\beta_- \gg 1$ and so $v_{\rm cut} \gg v_{\rm esc}$. In this case, capture is dominated by WIMPs that have velocities typical of the halo $v_h \sim 300\,{\rm km\,s^{-1}}$. This is the source of the "resonant enhancements" mentioned above: over the whole mass range $12\,{\rm GeV} \lesssim M_x \lesssim 65\,{\rm GeV}$, WIMP capture is dominated by resonances with elements common in the Earth, oxygen, magnesium, silicon, and iron. See Figure 2 from Gould (1987). For masses $M_x \gtrsim 65\,{\rm GeV}$, WIMP capture is overwhelmingly due to the "tail" of the iron resonance. If the velocity distribution at low velocities could be

approximated as a constant $f(\mathbf{u}) \simeq f(0)$, then the capture rate would take the form

$$C_0 = \frac{\pi N_A \sigma_{xA} f(0) v_{\rm cut}^4}{\beta_-} \to 4\pi N_A \sigma_{xA} f(0) v_{\rm esc}^4 \frac{M_A}{M_x} \qquad [\text{assuming } f(\mathbf{u}) = f(0) \text{ for } u < v_{\rm cut}]. \tag{3}$$

Thus in the high-mass limit (evaluated after the arrow), the capture rate would fall $C_0 \propto M_x^{-2}$: one power of M_x appears explicitly and the other is implicit in $f(\mathbf{u})$ for fixed ρ_x.

Gould (1988) showed that, unfortunately, the high-mass capture rate might not be so simple. If one restricts attention to WIMPs that are *not* bound tothe Sun, then

$$f_{\rm unbound}(\mathbf{u}) = 0 \qquad \text{for } |\mathbf{u} + \mathbf{v}_\oplus| \le 2^{1/2} v_\oplus, \tag{4}$$

where $\mathbf{v}_\oplus$ is the velocity of the Earth. To properly evaluate capture in the high-mass regime it is therefore necessary to evaluate the velocity distribution of bound as well as unbound WIMPs. In particular, if there were no bound WIMPs, then there would be no WIMP capture at all for WIMPs with $v_{\rm cut} \le (2^{1/2} - 1)v_\oplus$, and WIMP capture would be highly suppressed for WIMPs with $v_{\rm cut} \lesssim v_\oplus$. These thresholds correspond to $M_x \sim 320\,\mathrm{GeV}$ and $M_x \sim 120\,\mathrm{GeV}$ respectively.

Gould (1991) argued from detailed balance that regardless of the initial WIMP velocity distribution at the time of the formation of the solar system, $f_{\rm bound}(\mathbf{u})$ would be driven toward $f_\odot(0)$, the low-velocity limit of the velocity distribution in the solar neighborhood but away from the solar potential,

$$f_{\rm bound}(\mathbf{u}) \to f_\odot(0) \qquad \text{(Gould 1991)}. \tag{5}$$

He found that the charactersitic time for the evolution of the velocity distribution is less than the age of the solar system for $u \lesssim v_\oplus$ (Fig. 3 from Gould 1991). While these arguments still left indeterminate the bound WIMP distribution for $u \gtrsim v_\oplus$, in practice the capture rate would not be seriously affected for any mass even if all these orbits were empty. Hence Gould's (1991) arguments appeared to resolve the problem of WIMP capture for any M_x, σ_{xA} and Galactic WIMP distribution.

2. New Developments

There have been two new developments since 1991 that challenge this seemingly closed case. First, Farinella et al. (1994) have shown by direct numerical integration

that a large fraction of Earth-crossing asteroids are systematically driven into the Sun by various solar-system resonances. Gladman et al. (1997) and Migliorini et al. (1998) have further studied this problem and generally confirmed the initial results. These solar collisions occur on Myr time scales, several orders of magnitude faster than the characteristic diffusion times evaluated by Gould (1991). If WIMPs were, like asteroids, also driven into the Sun, they would be captured by the Sun and hence would be unavailable for capture by the Earth.

Second, Damour & Krauss (1998, 1999) have shown that WIMPs captured by the outer layers of the Sun into highly eccentric orbits could evolve into non-Sun-crossing orbits before they again collided with solar nuclei. If so, WIMPs on these eccentric orbits could substantially increase the number of low-velocity WIMPs in the solar neighborhood and so dramatically increase the capture rate in the mass range $60\,\mathrm{GeV} \lesssim M_x \lesssim 130\,\mathrm{GeV}$ (Bergstrom et al. 1999). Below 60 GeV, WIMP capture is dominated by the iron resonance while above 130 GeV, $v_{\rm cut} > v_\oplus$, the minimum speed relative to the Earth for WIMPs on highly eccentric orbits.

Here we assess the problem of interpreting neutrino-detection experiments in light of these two conflicting developments.

3. Basic Approach

The central experimental fact that must guide our analysis is that to date no neutrinos have been detected from the center of either the Earth or the Sun. The experiments therefore place upper limits on WIMPs. Thus, a conservative interpretation of these experiments requires that one assume that only those parts of velocity space are populated as can be justified based on very secure theoretical arguments. This perspective implies that we must assume that *all* bound WIMPs with Earth crossing orbits are driven into the Sun within a few Myr. This includes primordial WIMPs, the gravitationally diffused WIMPs of Gould (1991), and the solar-collision WIMPs of Damour & Krauss (1998, 1999).

Before continuing, we wish to emphasize that it is by no means proven that all bound WIMPs are in fact driven into the Sun. The numerical integrations carried out to date (Farinella et al. 1994; Gladman et al. 1997; Migliorini et al. 1998) apply to a very special subclass of Earth-crossing orbits, namely those of existing minor bodies. Near-Earth asteroids are believed to be transported from their reservoir in

the asteroid belt by means of resonances. It therefore may not be surprising that their continued orbital evolution is dominated by resonances. The main population of Earth-crossing WIMPs, which acquire their orbits by quite different, non-resonant paths (Gould 1991; Damour & Krauss 1998,1999), could be virtually unaffected by the resonances that drive asteroids into the Sun. Our viewpoint is simply that in the absence of proof that Earth-crossing WIMPs are not depopulated, one cannot place reliable upper limits on WIMPs from the failure to detect the annihilation signal that would be triggered by the capture of these Earth-crossing WIMPs.

4. Assured WIMP Capture

We begin by adopting an ultra-conservative view and assume that all bound WIMPs acquired by the solar system are immediately driven into the Sun. Then, for $v_{\rm cut} < (2^{1/2}+1)v_\oplus$, it is straight forward to show from equation (1) that

$$C_{\rm ultra} = \frac{2\pi N_A \sigma_{xA} f_\oplus(0)}{\beta_-}\int_{u=v_{\rm hole}-v_\oplus}^{v_{\rm cut}} du^2 (v_{\rm cut}^2 - u^2)\biggl(1 - \frac{v_{\rm hole}^2 - v_\oplus^2 - u^2}{2uv_\oplus}\biggr). \qquad (6)$$

where

$$v_{\rm hole} = 2^{1/2} v_\oplus. \qquad (7)$$

Hence, the ratio of the number of WIMPs captured under ultra-conservative assumptions to the naive formula (3) is

$$\frac{C_{\rm ultra}}{C_0} = \biggl\{\frac{1}{2}(1-s^2) + \frac{v_{\rm cut}}{v_\oplus}\biggl[-t^2(1-s) + \frac{(1+t^2)(1-s^3)}{3} - \frac{1-s^5}{5}\biggr]\biggr\}\Theta(1-s), \qquad (8)$$

where

$$s = \frac{v_{\rm hole}-v_\oplus}{v_{\rm cut}} = 0.41\frac{v_\oplus}{v_{\rm cut}}, \quad t = \frac{\sqrt{v_{\rm hole}^2 - v_\oplus^2}}{v_{\rm cut}} = \frac{v_\oplus}{v_{\rm cut}}, \qquad (9)$$

and Θ is a step function. The ratio $C_{\rm ultra}/C_0$ as a function of WIMP mass M_x is shown as a solid line in Figure 1.

Equation (6) is in fact too conservative. Gould (1991) showed that Jupiter-crossing orbits (including those that also cross the Earth's orbit) are populated from the reservoir of Galactic WIMPs on very short time scales. To an adequate approximation, the Jupiter-crossing orbits can be described as those with Earth-crossing velocities $\mathbf{u}$ constrained by $|\mathbf{u}+\mathbf{v}_\oplus| > [2(1-a_\oplus/a_J)]^{1/2}v_\oplus$, where $a_J/a_\oplus \simeq 5.2$

is the ratio of the orbital radii of Jupiter and the Earth. Hence the ratio of capture rates in this conservative (but not ultra-conservative) framework is still given by equation (8) but with $v_{\rm hole}^2 \to 2(1-a_\oplus/a_J)v_\oplus^2$ and consequently

$$s = 0.27\frac{v_\oplus}{v_{\rm cut}}, \quad t = 0.78\frac{v_\oplus}{v_{\rm cut}}. \tag{10}$$

This result is shown as a bold curve in Figure 1.

In principle one should take into account the loss of coherence in WIMP-nucleon interactions that occurs when the momentum transfer, q, becomes comparable to (or larger than) the inverse radius of the nucleus, $R \sim 3.7\,{\rm fm}$ for iron. The suppression due to loss of coherence is $\exp(-q^2R^2/3\hbar^2)$ (Gould 1987). However, for WIMPs of mass M_x, the maximum momentum transfer that leads to capture is $q_{\rm max} = (M_{\rm Fe}M_x)^{1/2}v_{\rm cut}$. In the high mass limit, $q_{\rm max} \to 2M_{\rm Fe}v_{\rm esc}$. Hence, the most extreme suppression factor is $\exp[-(2M_{\rm Fe}v_{\rm esc}R_{\rm Fe}/\hbar)^2/3] \sim 0.997$. We conclude that loss of coherence can be ignored. See also Gould (1987).

5. Discussion

From Figure 1, we see that under the conservative assumption that WIMPs do not populate bound orbits (unless they are Jupiter-crossing), WIMP capture is highly suppressed for WIMP mass $M_x \gtrsim 150\,{\rm GeV}$ and completely eliminated for $M_x > 630\,{\rm GeV}$. These results imply that, at present, the non-detection of neutrinos coming from the Earth's center cannot be used to place limits on WIMPs of mass $M_x > 630\,{\rm GeV}$. Moreover, for masses $75\,{\rm GeV} \lesssim M_x \lesssim 630\,{\rm GeV}$, the limits must be softened relative to what would be obtained from equation (3).

It is quite possible that WIMPs are not generically driven into the Sun. The evidence that they are comes from numerical integration of asteroid orbits, and these latter could occupy a very special locus in parameter space. It will be necessary to integrate typical WIMP orbits to find out if WIMPs survive longer than asteroids. In particular, these integrations should focus on the highly eccentric orbits for which Damour & Krauss (1998,1999) predict a huge enhancement for $M_x \lesssim 130\,{\rm GeV}$. If typical WIMP orbits are found to survive substantially longer than asteroid orbits, then the limits derived from the naive calculations of Gould (1987, 1991) would become valid. It is even possible that the much stronger limits derived by Bergstrom et al. (1999) would apply.

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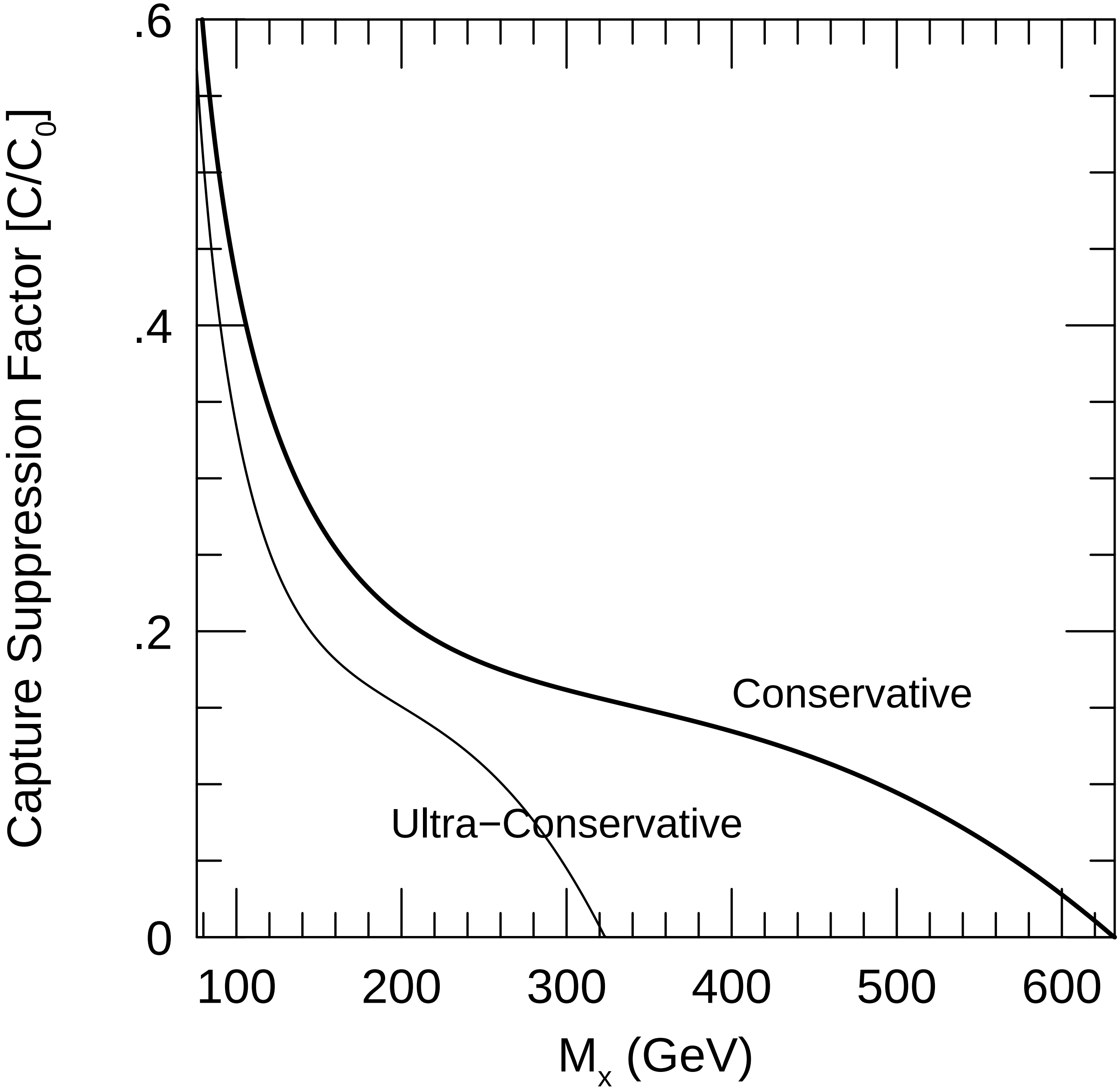
Fig. 1.— Conservative capture rates for WIMPs relative to the rate based on the naive assumption (Gould 1991) that the phase-space density of WIMPs bound to the Sun is similar to that of low-velocity unbound WIMPs. The solid curve shows the suppression factor under the ultra-conservative assumption that all bound WIMPs are driven into the Sun on short time scales (as is true of many Earth-crossing asteroids). The bold curve results from the more realistic assumption that WIMPs on Jupiter-crossing (and Earth-crossing) orbits are repopulated faster than they can be driven into the Sun.



.6
.4
.2
0
100
200
300
400
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600
Capture Suppression Factor [C/C$_0$]
M$_x$ (GeV)
Conservative
Ultra−Conservative